University of Texas MD Anderson Cancer Center
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Inequality Calculator, Version 3.1
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User's Guide

0. Overview

The purpose of the software is to calculate the probability that one random variable is greater than another. The two random variables are assumed to follow the same standard distribution family, with different parameter values. The distributions currently supported are: beta, gamma, inverse gamma, normal, log normal and Weibull.

The software may be downloaded from the web site:
http://biostatistics.mdanderson.org/SoftwareDownload/

This software occasionally sends usage statistics and crash reports to our biostatistics software support team to improve your experience using it.

License:

1. Description

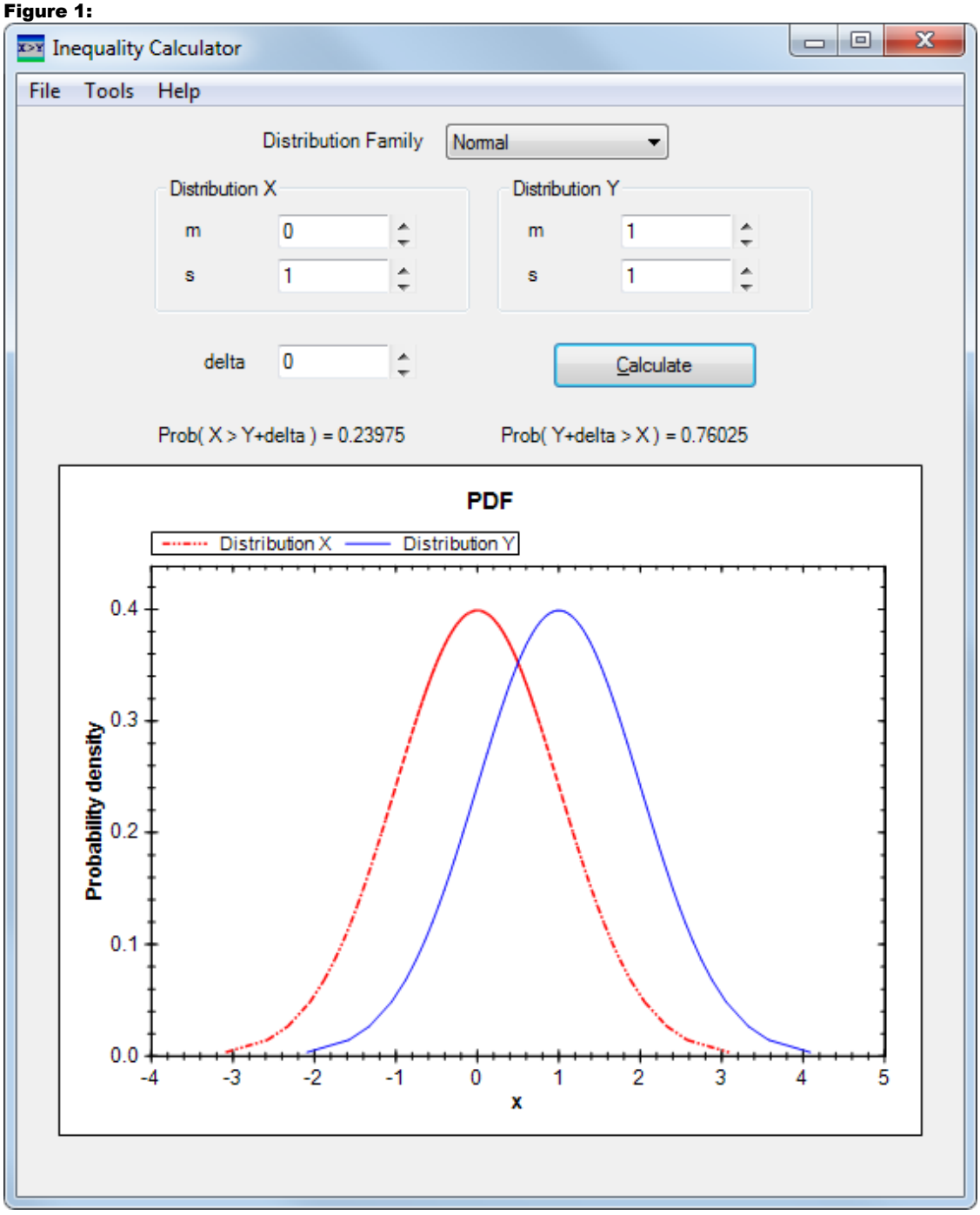
For two general continuous random variables X and Y, the probability that $X > Y$ is given by

$$P(X > Y) = \int_{-\infty}^{\infty} f_X(t) F_Y(t)\, dt$$

where f_X is the PDF (probability density function) of X and F_Y is the CDF (cumulative distribution function) of Y. The program computes this probability for the case of X and Y following the same standard distribution family.

In the example of Figure 1, we let $X \sim$ Normal(0, 1) and $Y \sim$ Normal(1, 1) and click on "Calculate" to obtain the result Prob($X > Y$) = 0.23975, along with the complementary value Prob($Y > X$) = 0.76025.

Figure 1:



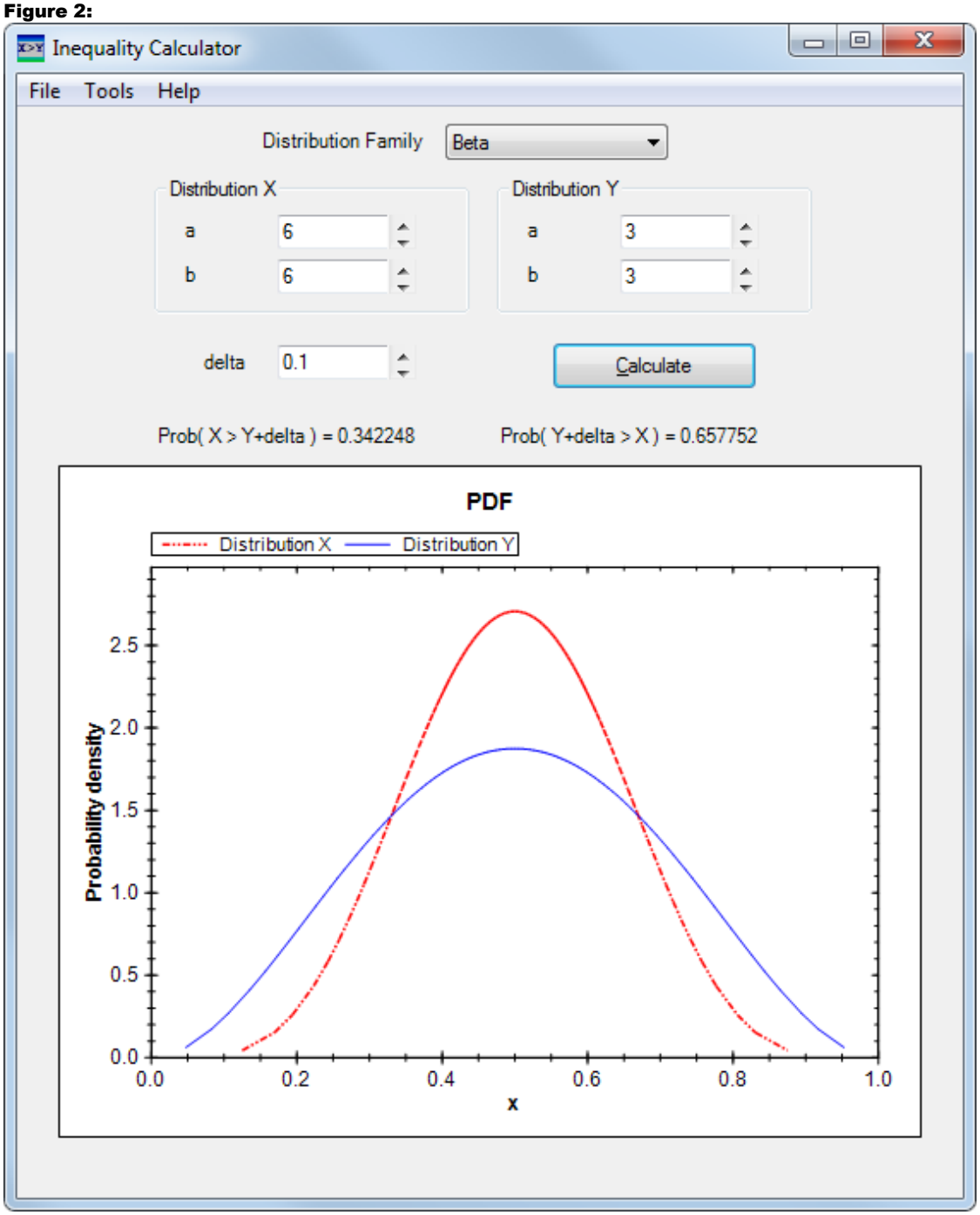
2. Shift Parameter

For the case of nonzero *delta* shift parameter, the probability inequality is given by

$$P(X > Y + delta) = \int_{-\infty}^{\infty} f_X(t) F_Y(t - delta)\, dt \ .$$

In the example of Figure 2, we let $X \sim$ beta(6, 6) , $Y \sim$ beta(3, 3) and $delta = 0.1$, giving the result 0.342248. If $delta = 0$ then the result would be 0.5 due to symmetry.

Figure 2:



3. Calculations

When the random variables X and Y follow one of the special distributions, there are more efficient methods for computing $P(X>Y)$ than directly applying the integration formula. For example, if X and Y follow exponential distributions (gamma with shape 1) with means μ_X and μ_Y, then

$$P(X > Y) = \frac{\mu_X}{\mu_X + \mu_Y}$$

If X and Y follow the normal distribution with parameters (μ_X, σ_X) and (μ_Y, σ_Y) respectively, then

$$P(X > Y) = \Phi\left(\frac{\mu_X - \mu_Y}{\left(\sigma_X^2 + \sigma_Y^2\right)^{1/2}} \right)$$

where Φ is the cumulative distribution function of the standard normal.

More information on these calculations can be found in the paper Numerical computation of stochastic inequality probabilities by John Cook.

For nonzero *delta* and the normal distribution family, $Y+delta \sim \text{Normal}(\mu_Y + delta, \sigma_Y)$ so that

$$P(X > Y + delta) = \Phi\left(\frac{\mu_X - \mu_Y - delta}{\left(\sigma_X^2 + \sigma_Y^2\right)^{1/2}} \right)$$

The other probability distribution families are not closed under translation (i.e., $Y+delta$ cannot be parameterized as a standard distribution when *delta* is nonzero), so the probability inequality calculations do use numerical integration.

4. Parameterizations

The software generally follows the conventions (with some exception) in the book *Statistical Distributions* by Merran Evans, Nicholas Hastings, and Brian Peacock.

4.1 Beta

The beta distribution with parameters a and b and has PDF

$$\frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)} x^{a-1}(1-x)^{b-1}$$

with mean

$$\frac{a}{a+b}$$

and variance

$$\frac{ab}{(a+b)^2(a+b+1)}$$

4.2 Gamma

The gamma distribution with shape parameter a and scale parameter b has mean ab, variance ab^2 and PDF

$$\frac{1}{\Gamma(a)b^a} x^{a-1}e^{-x/b}$$

4.3 Inverse Gamma

The inverse gamma distribution with shape parameter a and scale parameter b has PDF

$$\left(\frac{b^a}{x^{a+1}\Gamma(a)}\right)e^{-b/x}$$

If $a > 1$ then the mean is

$$\frac{b}{a-1}$$

If $a > 2$ then the variance is

$$\frac{b^2}{(a-1)^2(a-2)}$$

If X is distributed as a gamma distribution with parameters *(a, b)* then $1/X$ is distributed as an inverse gamma with parameters *(a, 1/b)*. Note that the b in our parameterization of the inverse gamma corresponds to *1/b* in another popular convention.

4.4 Normal

The normal distribution parameterized by its mean m and variance s^2 has PDF

$$\frac{1}{s\sqrt{2\pi}} e^{-\frac{1}{2s^2}(x-m)^2}$$

4.5 Log Normal

The log normal distribution is parameterized by m and s. If X is log normal with these parameters, $\log X$ is N(m,s). Note that m and s are *not* the mean and standard deviation of X but rather of $\log X$. The PDF is given by

$$\frac{1}{xs\sqrt{2\pi}} \exp\left(\frac{-(\log(x)-m)^2}{2s^2}\right)$$

with mean

$$\exp\left(m+\frac{1}{2}s^2\right)$$

and variance

$$\exp(2m+s^2)(\exp(s^2)-1)$$

4.6 Weibull

The Weibull distribution has a shape parameter a and scale parameter b. It has PDF

$$\frac{ax^{a-1}}{b^a} \exp(-(x/b)^a)$$

with mean

$$b\Gamma((a+1)/a)$$

and variance

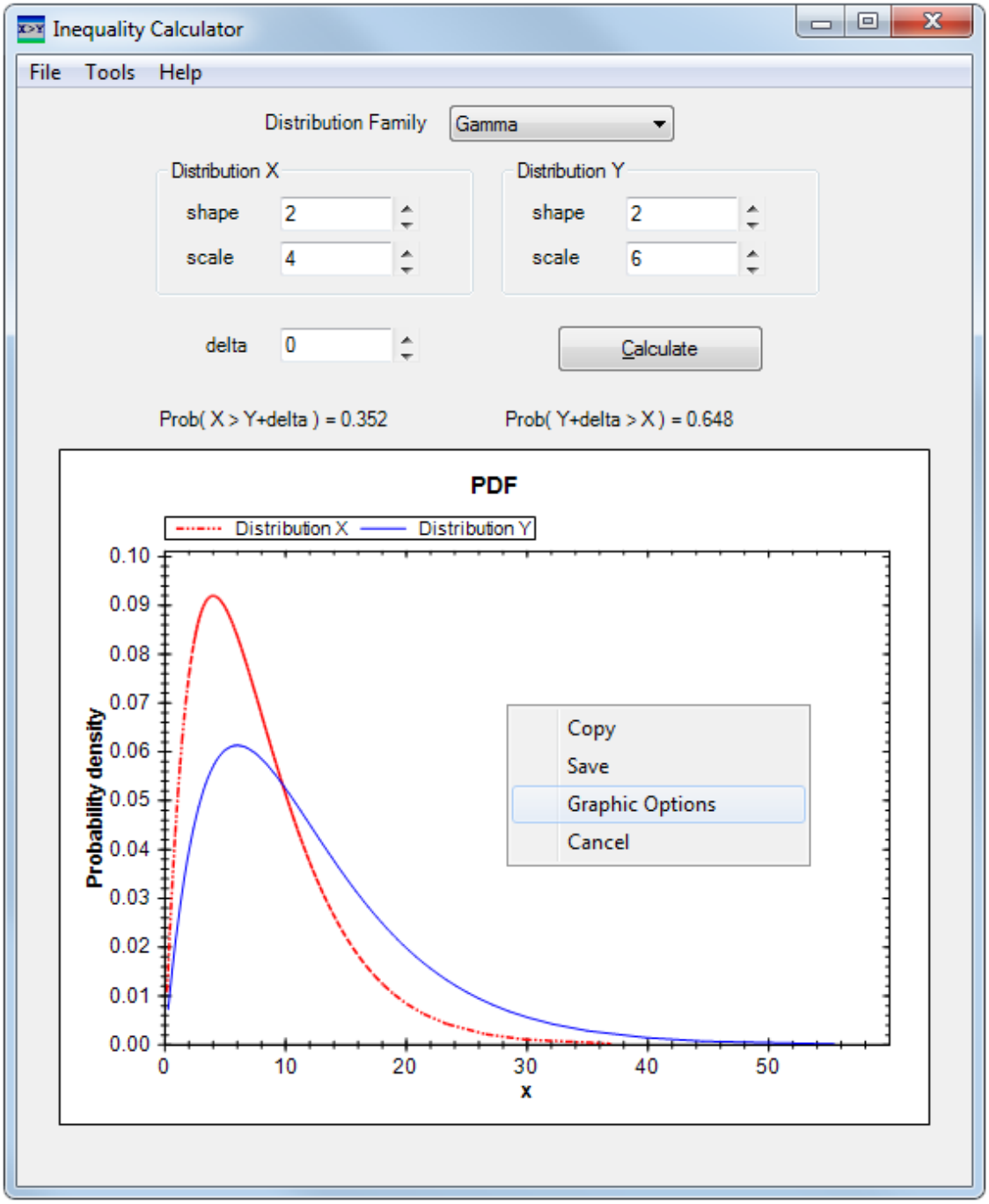
$$b^2\left(\Gamma((a+2)/a)-\Gamma((a+1)/a)^2\right)$$

5. Miscellaneous

5.1 Graphic Options

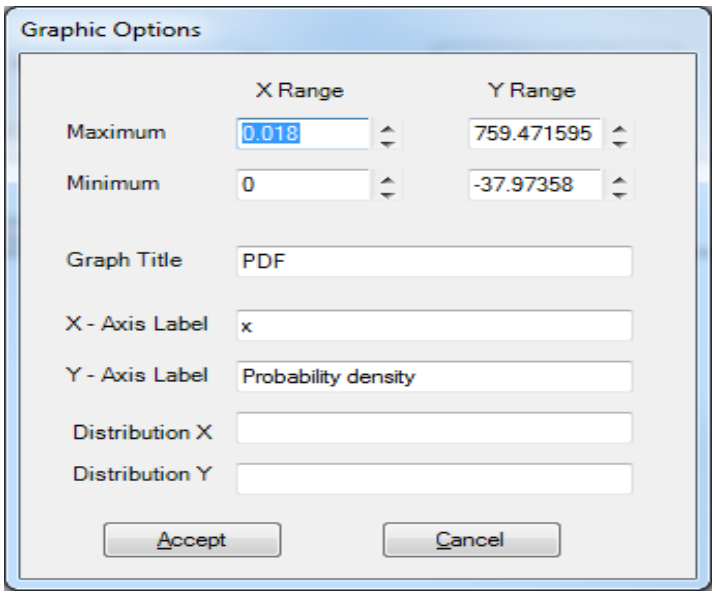
The user may customize the PDF plot by right-clicking on the window to bring up a menu with the item “Graphic Options” (Figure 5a).

Figure 5a:



Choosing this item will bring up the dialog box in Figure 5b. Any change to one of these dialog fields will be reflected immediately in the graph pane. Clicking the “Accept” button will save changes to the graph, while clicking “Cancel” will discard all changes.

Figure 5b:



The "Copy" menu item in the popup menu of Figure 5a will copy the graph image to the clipboard where it can be pasted into another document.

The "Save" menu item will bring up a "Save As..." dialog box for selecting the name and location of the file to be saved. The quality of the graph is dependent on the size of the graph when it is copied or saved. Therefore, it is suggested that the window be maximized before copying or saving the graph. This will produce a clearer image when enlarged.

5.2 Error Indicator

Each numerical input field is equipped with an error indicator. When the symbol is displayed, its adjacent field contains an invalid entry. By moving the mouse cursor over this symbol, a tip will be displayed as shown in Figure 5c.

Figure 5c: